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# Saturation effects at LHC energies

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Within the framework of a modified Balitsky-Kovchegov equation, we calculate and provide estimates of non-linear saturation effects expected in the LHC range of energies.

The main objective of this letter is to provide reliable estimates for high density QCD (saturation) effects at the LHC range of energies. Our estimates are based on the Balitsky-Kovchegov [1] equation which is the mean field approximation to high density QCD [2, 3, 4]. In spite of the restricted theoretical accuracy of this equation [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] in the framework of the colour dipole approach [18], the physics underlying it is rather transparent, and it has led to a successful description of the main features of the experimental data, both for deep inelastic scattering and ion-ion collisions [19, 20, 21, 22, 23]. Among the advantages of the Balitsky-Kovchegov equation is the theoretical understanding of its mathematical structure [2, 24, 25], the analytical solutions in the restricted domains [25, 26, 27], as well as extensive experience in numerical solutions [28, 29, 30, 31, 32, 33, 34, 35, 36]. This equation is at the moment the best that we have.

However, the Balitsky-Kovchegov equation has very serious deficiencies: it does not include the next-to-leading corrections to the BFKL equation [39] which are large [40], and those should be

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taken into account to obtain reliable predictions at higher energies [41]. In the estimates given here we rely on the modified version of the Balitsky-Kovchegov equation which was suggested in Ref. [42]. The modified Balitsky-Kovchegov equation includes the correct re-summation of the NLO corrections to the BFKL equation suggested by the Florence group [43]. It also contains the important observation of the Durham group[44], that the main part of the NLO correction is the inclusion of the leading order anomalous dimension of the DGLAP equation [45], this in the framework of the BFKL approach.

The modified Balitsky-Kovchegov equation suggested[49] is

$$\begin{aligned} \frac{\partial N(r, Y; b)}{\partial Y} &= \frac{C_F \alpha_S}{\pi^2} \int \frac{d^2 r' r^2}{(\vec{r} - \vec{r}')^2 r'^2} \left( 1 - \frac{\partial}{\partial Y} \right) \left[ 2N\left(r', Y; \vec{b} - \frac{1}{2}(\vec{r} - \vec{r}')\right) - \right. \\ &\quad \left. - N\left(r, Y; \vec{b}\right) - N\left(r', Y; \vec{b} - \frac{1}{2}(\vec{r} - \vec{r}')\right) N\left(\vec{r} - \vec{r}', Y; b - \frac{1}{2}\vec{r}'\right) \right] \end{aligned} \quad (1)$$

Eq. (1) includes the full anomalous dimension of the DGLAP equation in leading order, in the approximate form proposed in Ref. [46], namely,

$$\gamma(\omega) = \bar{\alpha}_S \left( \frac{1}{\omega} - 1 \right) \quad (2)$$

which describes the exact  $\gamma(\omega)$  within an accuracy  $\leq 5\%$ . The advantage of Eq. (1) is that it conserves energy even when including the non-linear term.

### **Unintegrated structure function:**

Solving Eq. (1) we obtain the dipole scattering amplitude. The more transparent physical meaning is the so called unintegrated structure function, which is the probability for a hadron to have a gluon with fixed transverse momentum. This function enters the calculation of the inclusive production for the gluon jet, as well as all other inclusive cross sections. [2, 37, 38],

$$\frac{d^2 \sigma}{dy d^2 p_t} = \frac{4 N_c \alpha_S}{N_c^2 - 1} \frac{1}{p_t^2} \int \phi(Y - y, k_t) \phi(y, \vec{p}_t - \vec{k}_t) d^2 k_t \quad (3)$$

where the unintegrated structure function  $\phi$  is determined from the solution of Eq. (1) by the following equation [38]

$$\phi(Y, k_t) = \frac{N_c}{4 \alpha_S(k_t^2) \pi^2} k_t^2 \int d^2 b \Delta_{k_t} N(Y; k_t, b) \quad (4)$$

where  $\Delta_k$  is a two dimensional Laplacian and

$$N(Y; k_t, b) = \int \frac{d^2 r}{(2\pi)^2 r^2} e^{i\vec{k}_t \cdot \vec{r}} N(Y; r, b). \quad (5)$$

and the gluon structure function is equal to

$$\alpha_S(Q^2) xG(x, Q^2) = \int^{Q^2} d k_t^2 \alpha_S(k_t^2) \phi(Y, k_t) \quad (6)$$

### Suppression due to the saturation effects:

In Fig. 1 we plot the ratio

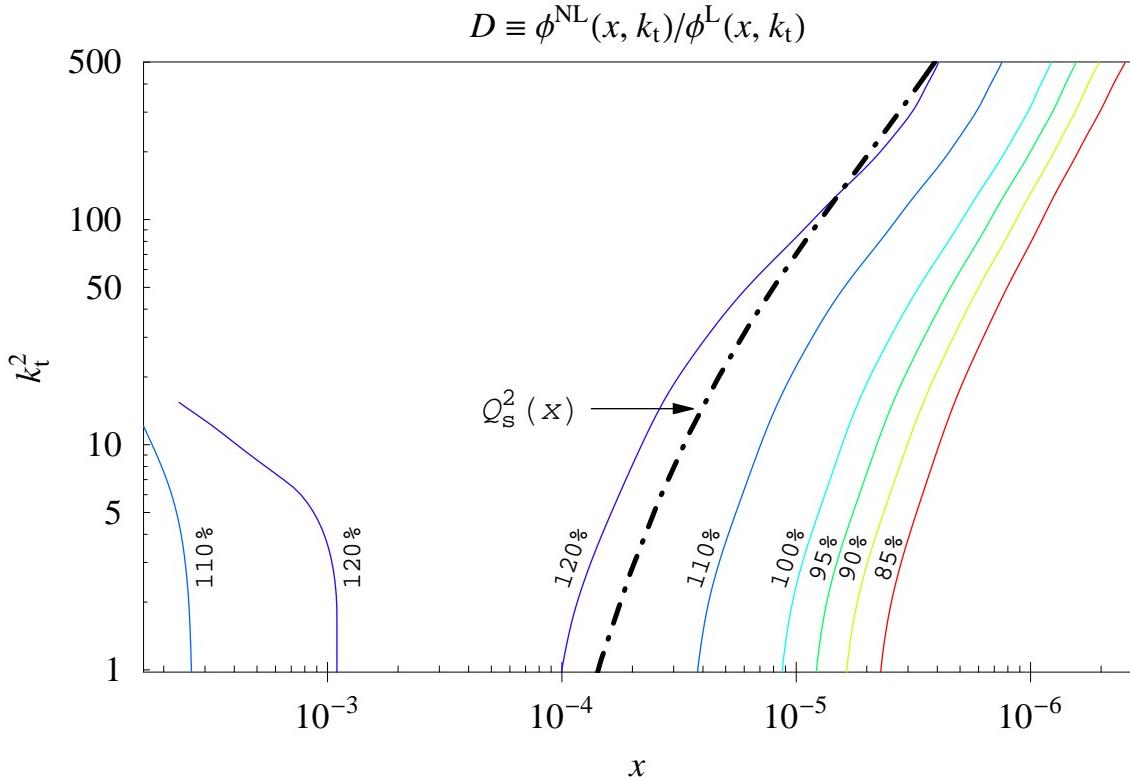
$$D(k_t; x) = \frac{\phi^{NL}(k_t, x)}{\phi^L(k_t, x)} \quad (7)$$

where  $\phi^{NL}$  is the unintegrated structure function obtained using all terms of Eq.(1), while  $\phi^L$  is, the unintegrated structure function obtained by excluding the non-linear part of Eq.(1).

Our initial condition for solving Eq. (1), are the dipole-proton amplitude from the CTEQ parameterization for the gluon structure function. We use the following form of this initial distribution

$$N(r, x; b) = 1 - \exp\left(-\frac{\alpha_S \pi^2}{6} r^2 x G^{CTEQ(6)}(x, 4/r^2) S(\sqrt{r^2 + b^2})/S(0)\right) \quad (8)$$

where  $S(b)$  is a dipole-like profile,  $R$  is the size of the proton, we took  $R^2 = 10 \text{ GeV}^{-2}$ .



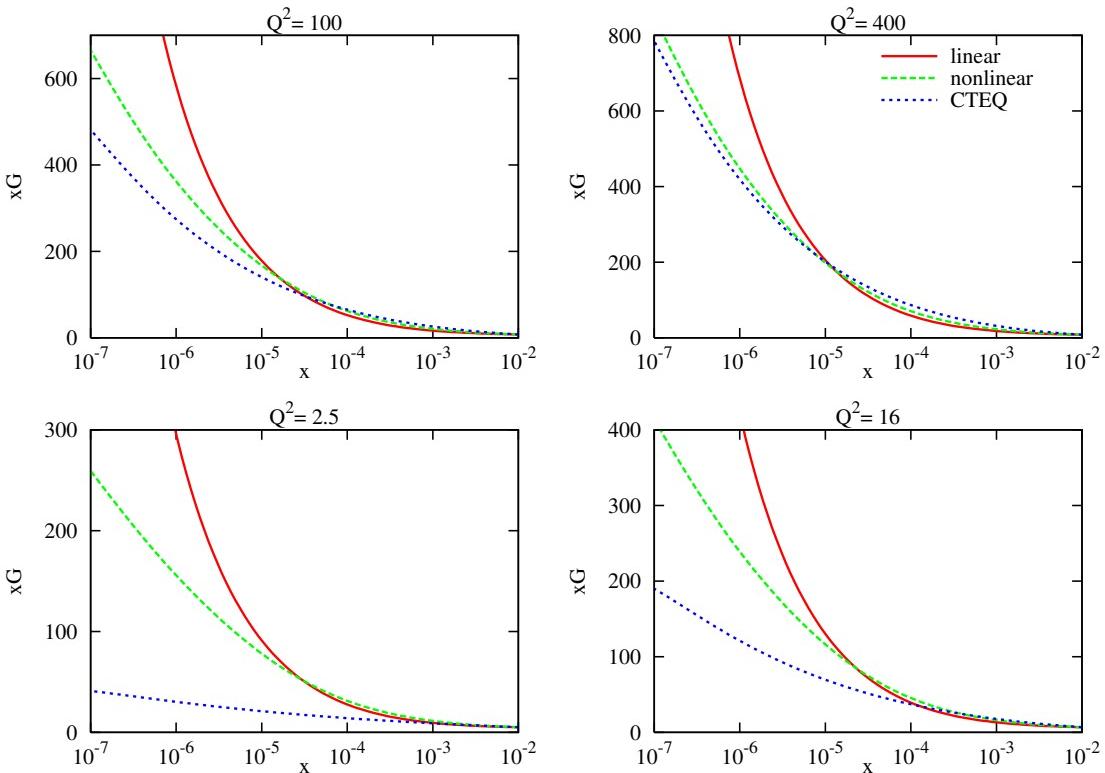
**FIG. 1:** The ratio given by Eq. (7) which shows the effect of the non-linear terms on the prediction for the dipole scattering amplitude at the LHC energies.

From Fig. 1 one can see that the non-linear term in Eq. (1) is important, and in the LHC range of energy it suppresses the value of  $\phi$  by 30%. It should be stressed that the inclusive cross section is proportional to  $\phi^2$ , and we expect even a larger suppression for the inclusive production (approximately twice as large). Note that  $D(x, k_t) < 1$  for  $k_t^2 < Q_s^2$ . For  $k_t^2 \approx Q_s^2$ , on the other hand, we observe an antishadowing effect on which we shall comment below [50]. Such behaviour of the ratio was expected theoretically (see for example Refs. [19, 20, 21, 27]), however there are

no reliable estimates in the approach which is based on the solution of the Balitsky-Kovchegov equation, and which describes all data at lower energies.

### Increase due to the saturation effects (antishadowing):

Fig. 2 shows the value of  $xG(x, Q^2)$  for the non-linear equation (see Eq. (1)), together with the value for the solution of the linear equation (Eq.(1) without the non-linear terms). Note that the solution to the non-linear equation leads to a larger amplitude for large values of  $x$ . This is not expected as the non-linear term has a negative sign, and manifests itself as the shadowing correction which decreases the value of the amplitude. The increase of the ratio  $D$  (see Eq. (7) and Fig. 3) at low energies is significant, and cannot be explained by errors due to the approximate formula for the anomalous dimension of the DGLAP equation, or due to the numerical procedure.



**FIG. 2:** The value of the gluon structure function as function of  $x$  for different values of  $Q^2$ .

Non-linear denotes the curve for the gluon structure function calculated using Eq. (6) from the solution of Eq. (1), while linear is the same without the non-linear term. CTEQ denotes the curve for the gluon structure function given by CTEQ parameterization.

The explanation for the anti-shadowing effect is very simple. At large  $k_t$ , the non-linear term is small and the linear evolution equation is a good approximation to the solution. Decreasing the value of  $k_t$ , the largest changes are induced in the slope of the amplitude (or/and gluon densities) with respect to  $\ln(k_t)$  (see Refs. [2, 20, 21, 27]). The slope for the linear equation is larger than

for the non-linear equation. For large values of  $k_t$ , the unintegrated structure function  $\phi$  is equal to the solution of the linear equation, while for small values of  $k_t$ ,  $\phi$  becomes larger than the solution to the linear equation.

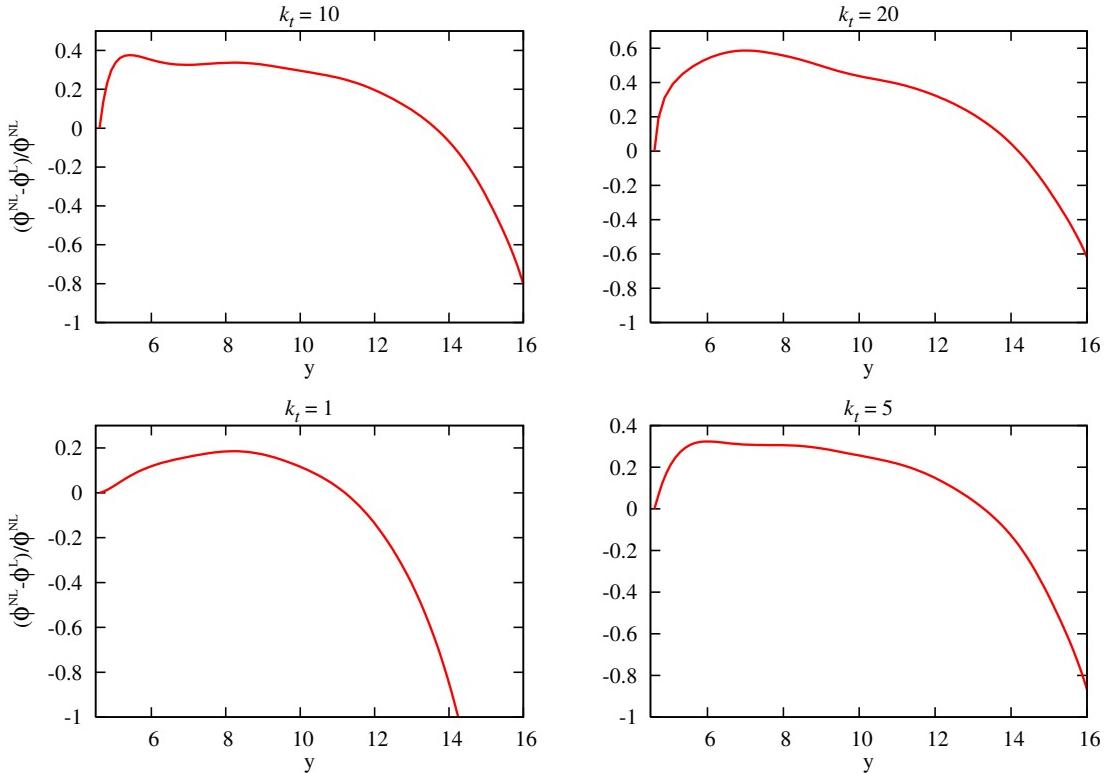
Another way to see the same effect is by comparing derivatives with respect to  $Y$  see Fig. 2. For the solution to the non-linear equation at large  $x$  the derivatives with respect to  $Y$  are much larger than the same derivative for the solution of the non-linear equation. Therefore, for the case of the non-linear equation the linear term, which contains  $-\partial/\partial Y$ , is larger for the non-linear equation, leading to an increase in the value of the solution. The unintegrated structure function in Eq. (4) reproduces the same qualitative behaviour as the dipole scattering amplitude.

All these arguments can be obtained directly from energy-momentum sum rules which hold for the modified Balitsky-Kovchegov equation, namely, that the integral over  $x$  is independent of  $Q^2$

$$\int dx \phi^{NL}(Y; Q^2) = Const(Q^2) = \int dx \phi^L(Y; Q^2) \quad (9)$$

In the region  $k_s^2 < Q_s^2(x)$  (or  $x < x_s$ , where  $x_s$  is the solution of the equation  $k_t^2 = Q_s^2(x_s)$ ), we expect  $D(x, k_t) < 1$  (see Fig. 2) and based on Eq. (7), we anticipate that in some region of  $x > x_s$ , this ratio should be larger than 1.

Fig. 3 displays quantitative estimates for these antishadowing effects.



**FIG. 3:** The ratio  $(\phi^{NL} - \phi^L)/\phi^{NL}$  for different values of  $Q^2$  as function of  $y = \ln(1/x)$ .

The effect is rather large and has been discussed previously in Ref. [47]. In our approach it arises naturally [51].

The comparison of the results of the present calculation with those of CTEQ and MRST is not straight forward, since we calculated the unintegrated gluon distribution, while CTEQ and MRST present results only for the structure functions. We are presently busy with this project and hope to be able to present our results in the near future.

We hope that our estimates of the scale for the nonlinear effects at the LHC energies, will help to produce more reliable calculations for the cross sections of ‘hard’ processes of interest at high energies.

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- [49] We have solved a slightly different equation (see Eq.(3.1) in Ref. [42]) The equation, that has been solved, is a simplified version of Eq. (1) due to numerical difficulties.
- [50] We define the saturation momentum as the value of  $Q = 2/r$  at which  $2 \log(1 - N(Y, r; b = 0)) = -1$ . In order to compare  $Q_s^2$  with the ratio  $D(x, k_t)$ , which is an integrated quantity, we have normalized  $Q_s^2$  such that at the leftmost point, it coincides with the saturation momentum of the GBW model [19].
- [51] In the talk of K. Peters at HERA-LHC Workshop (19 January 2005), it was claimed that there is no antishadowing effect. As far as we understood from his transparencies, he used the modification to the Balitsky-Kovchegov equation suggested in Ref. [48]. This modification is quite different from ours and does not preserves energy conservation in both the linear and non-linear terms. This is the reason why no anti-shadowing effect was seen.